Monte Carlo UQ for Control Systems With Convex Optimization

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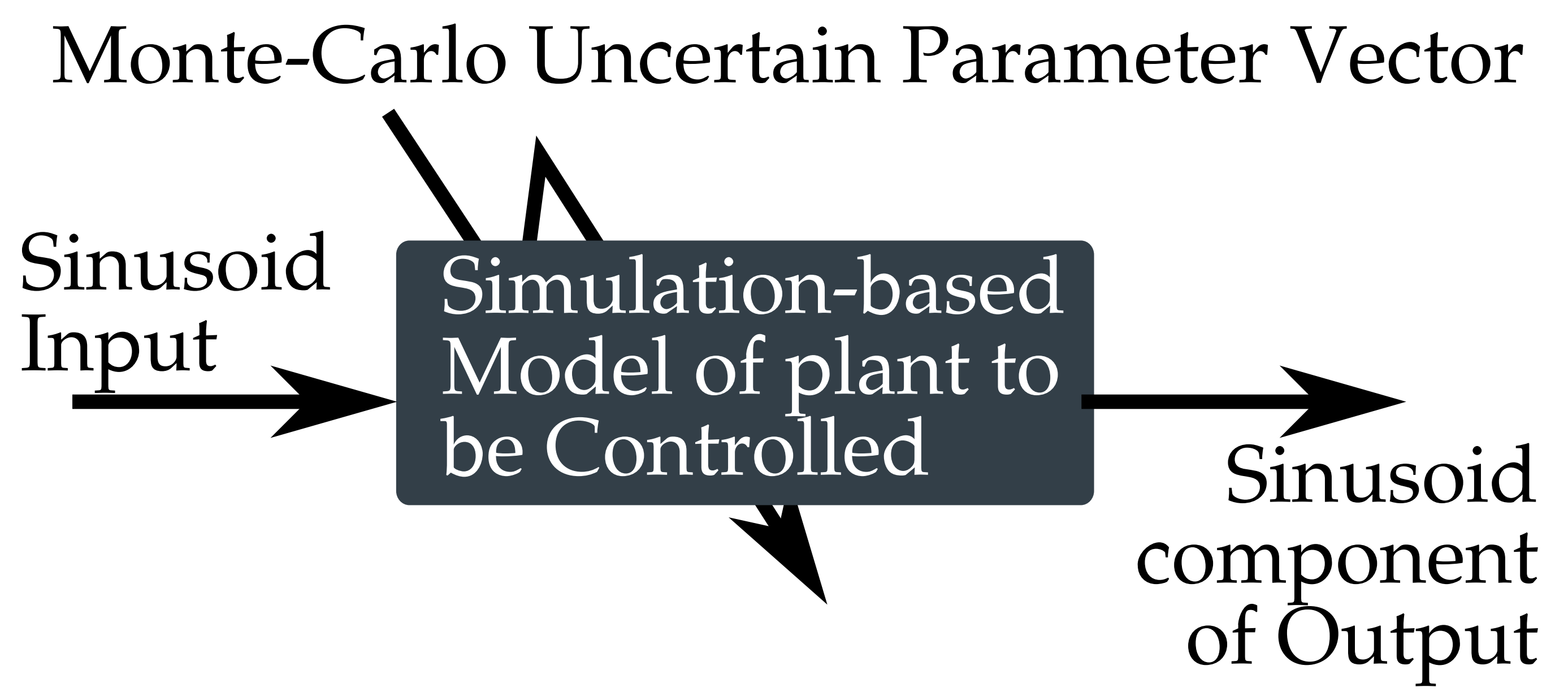
Introduction

We seek to quantify uncertainty using the model from robust feedback control theory. This frequency domain model makes strong guarantees about the stability of feedback controllers, and so has application in robotics, and simulation for robotics.

Monte Carlo Tests

Frequency domain models map sinusoid inputs to sinusoid outputs in a frequency dependant way. But this relationship may depend on various unknown factors. By randomly selecting those factors we can see a single possible input output pair.

Generation of a Single Data Point



Monte-Carlo Uncertain Parameter Vector

Sinusoid Input

Simulation-based Model of plant to be Controlled

Sinusoid component of Output

The "H∞" Model

Robust Control Theory extends the following linear model:

$$Y(j\omega) = \mathbf{P}(j\omega)U(j\omega)$$

where Y is output, P is plant, and U is input all in Frequency Domain. It adds an uncertainty term

$$Y(j\omega) = \left[\mathbf{P}(j\omega) + E(j\omega)\Delta J(j\omega)\right]U(j\omega)$$

in which the Delta object is bounded in magnitude, and E and J describe weighting matrix functions that determine the shape of the uncertainty.

A "Cone Parameterization"

We have cleverly re-written the H infinity model as a quadratic form inequality for any particular frequency:

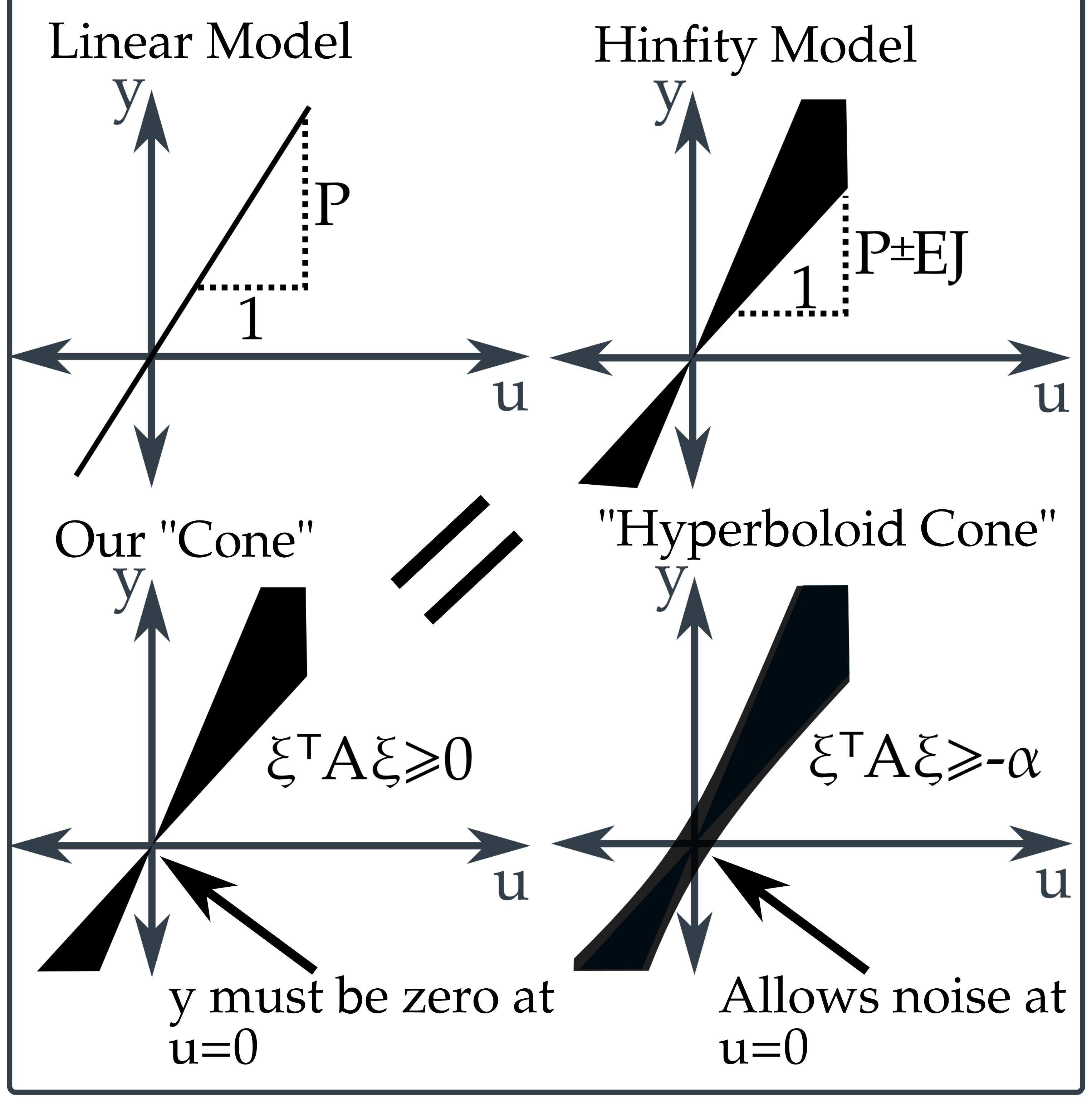
$$\begin{bmatrix} Y \\ U \end{bmatrix}^H \mathbf{A} \begin{bmatrix} Y \\ U \end{bmatrix} \geq 0$$

with the block matrix

$$\mathbf{A} = \begin{bmatrix} -\mathbf{E}^{-H}\mathbf{E}^{-1} & \mathbf{E}^{-H}\mathbf{E}^{-1}\mathbf{P} \\ \mathbf{P}^H\mathbf{E}^{-H}\mathbf{E}^{-1} & \mathbf{J}^H\mathbf{J} - \mathbf{P}^H\mathbf{E}^{-H}\mathbf{E}^{-1}\mathbf{P} \end{bmatrix}$$

At any frequency we can use convex optimization to choose a constant matrix A which satisfies this cone inequality for all the measured data points. If we expect some noise, we can soften this inequality with a small bias alpha. We call the resulting shape a "hyperboloid cone".

A 2D Analogy:



Linear Model

Hinfity Model

Our "Cone"

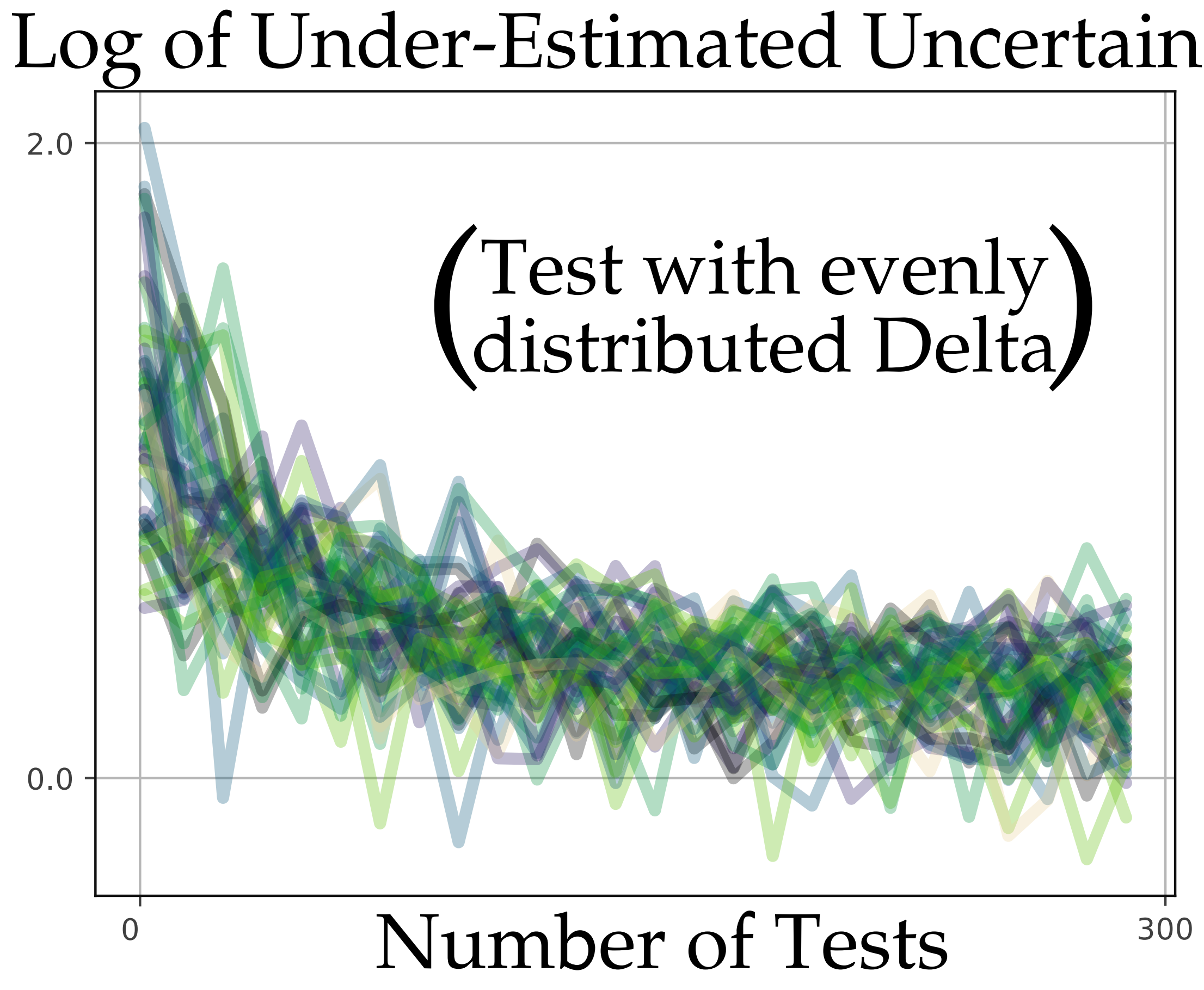
"Hyperboloid Cone"

$\xi^T A \xi \geqslant 0$

$\xi^T A \xi \geqslant -\alpha$

y must be zero at u=0

Allows noise at u=0



Log of Under-Estimated Uncertainty

Conclusion

Directly testing models under random conditions is a plausible method of obtaining H infinity control models. Fitting models to data is convex, at least within a single frequency. Uncertainty quantification for control applications has exploitable structure. Only the extreme-most results matter in the end.

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